Mathematics - Course 421

GRAPHING FUNCTIONS

I Introduction to Functions

Definition:

One variable is a *function* of another variable if a unique value of the first variable corresponds to each value of the other, ie, if the two variables are related by some formula (loosely speaking).

Notation:

The notation f(x), A(r), P(T), etc is used to denote f as a function of x, A as a function of r, P as a function of T, etc.

Example 1:

The area A of a circle is a function of its radius r according to the formula,

$A(r) = \pi r^2$ (read "A at r equals πr^2")

ie, a definite value of A corresponds to each value of r

eg, $A(1) = \pi(1)^2 = 3.14$

$A(5) = \pi(5)^2 = 78.5$

$A(0.1) = \pi(0.1)^2 = 0.0314$

etc.

Example 2:

$f(x) = x^3 - 5x$ (read "f at x equals $x^3 - 5x$")

Here f is a function of x since the formula gives a unique value of f for each value of x

eg, $f(0) = 0^3 - 5(0) = 0$

$f(1) = 1^3 - 5(1) = -4$

$f(-2) = (-2)^3 - 5(-2) = 2$

etc.

August 1978

Functions of Several Variables:

If G is a function of n variables, x_1, x_2, ..., X_n) one writes

$$G(x_1, x_2, \ldots, X_n)$$

Example 3:

Cylinder volume V is a function of both height h and radius r, according to the formula,

$$V(r, h) = \pi r^2 h$$

ie, each pair of r and h gives a unique volume

eg, $V(1, 1) = \pi(1)^2\ (1) = 3.14$

$V(2, 5) = \pi(2)^2\ (5) = 62.8$

etc.

Dependent and Independent Variables:

The *independent variable* is the one to which values are assigned arbitrarily, and the *dependent variable* is the one given by the formula.

eg,

dependent variables

$$\overset{\downarrow}{A}(\underset{\uparrow}{r}); \qquad \overset{\downarrow}{f}(\underset{\uparrow}{x}); \qquad \overset{\downarrow}{G}(\underset{\uparrow}{x_1}, \underset{\uparrow}{x_2}, \ldots, \underset{\uparrow}{X_n})$$

independent variables

II Graphing Functions

Usually the independent variable is plotted along the x-axis (horizontally) and the dependent variable along the y-axis (vertically) - cf 421.40-1, part III.

The steps to graphing a function are similar to those outlined in § 221.40-1, part III for data graphs, with the following notable differences:

(1) The table of values must be calculated, using the function relationship.

(2) The plotted points are <u>always</u> joined by a smooth curve (except for discontinuous functions, which are beyond the scope of this text).

(3) The curve is labelled with the equation which it represents.

Example 1:

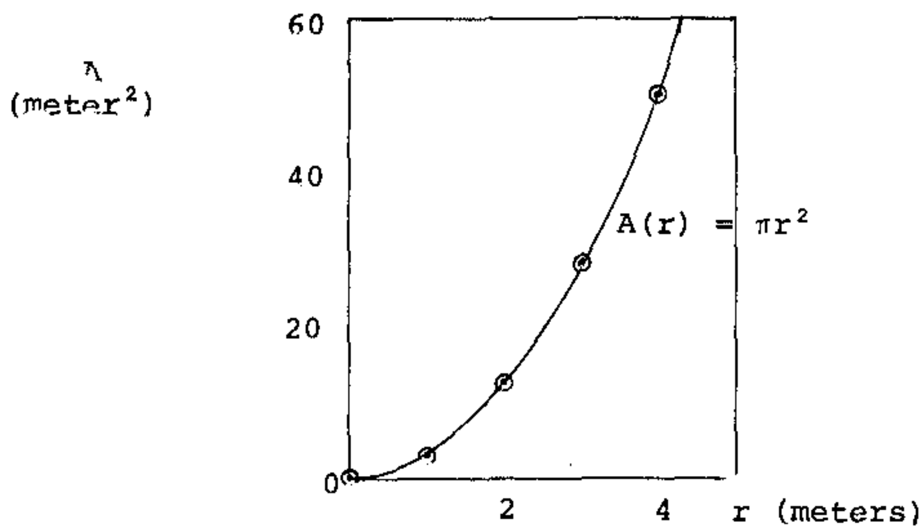
Plot a graph showing circle area A as a function of radius r in meters, $0 \leq r \leq 4$.

Solution:

Use $A(r) = \pi r^2$ to generate a table of values.

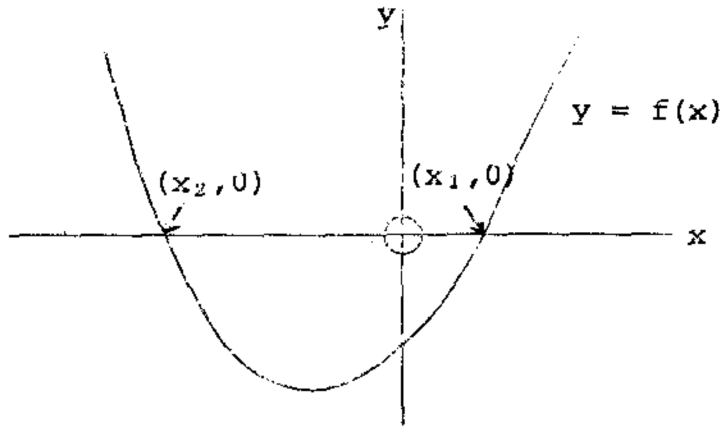
r meters	0	1	2	3	4
A(r) meters^2	0	3.1	12.6	28.3	50.3

Graph of Circle Area vs Radius



Roots of an Equation:

The *roots* of any equation of the form $f(x) = 0$ are the x values which satisfy this equation (make it true). Clearly, the x-coordinates of the x-intercepts of the curve $y = f(x)$ are the roots of $f(x) = 0$, as illustrated below:



x_1, x_2 are the roots of $f(x) = 0$

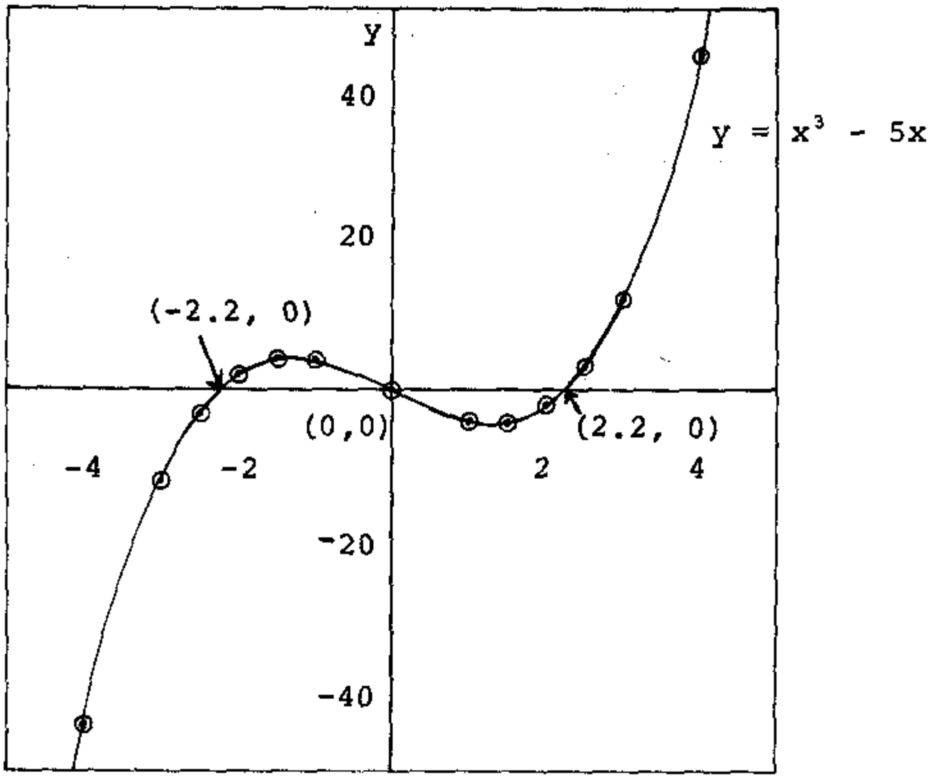
Example 2:

Graph the function $f(x) = x^3 - 5x$ and find the roots of $x^3 - 5x = 0$ from the graph.

Solution:

Let $y = f(x)$, and use $y = x^3 - 5x$ to generate a table of values

x	0	± 1	± 1.5	± 2	± 2.5	± 3	± 4
y	0	∓ 4	∓ 4.1	∓ 2	± 3.1	± 12	± 44



Roots of $x^3 - 5x = 0$ are $x = \pm 2.2$ and $x = 0$

ASSIGNMENT

1. Express each of the following statements in functional notation, and give the exact formula for the notation:

   (a) The circumference C of a circle is a function of its radius r.

   (b) The distance d travelled in time t at a uniform speed v is a function of t and v.

   (c) The total area A of the surface of a right circular cylinder is a function of its height h and radius r of its base.

2. Given $f(x) = 2x - 3$, find $f(6)$, $f(0)$, $f(-2)$.

3. Given $H(x) = x(x - a)(x - 1)$ find $H(0)$, $H(1)$, $H(a)$.

4. Find the length d of a diagonal of a square as a function of the perimeter p of the square.

5. Graph the following functions $f(x)$ and find the roots of $f(x) = 0$ from the graphs:

   (a) $4 - x^2$

   (b) $x^2 + 2x + 2$

   (c) $2 + 9x - x^3$

   (d) $x^2 - x - 6$

   (e) $x^3 - 3x - 1$

L. Haacke